Prediction of thermal fatigue life of ceramics

N. KAMIYA, O. KAMIGAITO
Toyota Central Research and Development Laboratories, Inc, 2–12, Hisakata, Tempaku-ku, Nagoya, 468 Japan

On the assumption that the thermal fatigue life of ceramics is determined mainly by the duration over which a crack reaches a small critical length, a prediction of the life was made by application of fracture mechanics to ceramics based on subcritical crack growth. Approximated formulae, such as $N/N' \simeq (\Delta T_{N'}/\Delta T_N)^n$, were derived, where N and N' are the lives for temperature differences of ΔT_N and $\Delta T_{N'}$, respectively, and n is a material constant. Experimental examination showed that the formulae proved to be valid for glass, sintered mullite under moderate shock severity, and zirconia. Data given by other authors also prove their validity. The deviation of the life from the formulae for sintered mullite under a thermal shock of extremely low severity, suggests that a certain mechanism, for example strengthening, is needed to understand the life of the sintered mullite. Moreover, n determined from the thermal shock experiments is comparable with that given in data determined by a mechanical method.

1. Introduction

Recent development in the fracture mechanics of ceramics leads to the prediction of their static fatigue life. For the prediction of thermal fatigue life, however, tedious numerical calculations are needed [1], which are not so easy as that for static fatigue life. This seems to follow from the fact that the thermal stress distribution over the duration of thermal stressing cannot be known precisely.

Consideration of the change in crack growth rate with crack length shows that the fatigue life can be approximated by the duration over which the crack length reaches a certain critical value, beyond which the growth becomes relatively rapid. Thus, for a rough estimation of the life, only knowledge of the stress distribution in a restricted region of a ceramic body is needed. From such an approximation, the prediction may be made rather easily.

2. Theory

The slow crack growth of ceramics can be described by the following equation [2],

$$da/dt = A \cdot \exp(-Q/RT)K_I^n \qquad (1)$$

where a, t, T, K_I, R, Q, A and n are crack length, time, temperature, stress intensity factor, gas constant and material constant, respectively. n is $\geqslant 20$ for most ceramics.

K_I may also be expressed as

$$K_I = Y \cdot \sigma \cdot a^{1/2} \qquad (2)$$

where Y and σ are the geometrical factor and the applied stress, respectively [3].

Substitution of Equation 2 into Equation 1 gives

$$da/dt = A \cdot \exp(-Q/RT)Y^n \cdot \sigma^n \cdot a^{n/2}. \quad (3)$$

In general, the stress, σ, consists of mechanical stress, σ_M, and thermal stress, σ_T. When thermal fatigue determines the life of ceramics, σ_M can be neglected and σ_T is used as σ in Equation 3. Under thermal stress, generally, the stress distribution and type of loading change with crack length as well as time, depending on the shape of the ceramic specimens and the heating and cooling processes. Thus, in general, Y and σ depend on the crack length as well as time and temperature. T also changes with time and crack length. This makes it difficutl to obtain an analyti-

cal solution for a in Equation 3. For the prediction, however, the following approximation can be made. In most cases, σ as well as Y is not such a rapidly decreasing function of a, that the growth rate can be expected to become very rapid as the crack grows. Therefore, a substantial part of the fatigue life is expected to have passed before the crack length reaches a certain critical value, a_c.

The critical length is generally thought to be small. For example, for a ceramic of $n = 20$, assuming that Y and σ are not rapidly decreasing functions of a, the ratio of the crack velocity, v_1, at $a = 10\,\mu m$, to v_2, at $a = 20\,\mu m$, is about 10^{-3}. The time (or the number of cycles of repeated thermal shocks), t_1, for the crack to reach a depth of $10\,\mu m$ from the surface, is much longer than $10\,\mu m/v_1$. The time, t_2, for the crack to reach a depth of $10^4\,\mu m$ from that of $20\,\mu m$, is much shorter than $10^4\,\mu m/v_2$. Therefore, $t_1 \gg t_2$, which shows that the critical length can be taken to be of the order of $10\,\mu m$. Variation of σ and Y would not change the value substantially.

As seen above, it can be assumed that the crack growth for a very small range of a would determine the life. Thus, in Equation 3, Y, σ_T and T can be approximated to be independent of a. It also implies that Y, σ_T and T are independent of the number of repeated thermal shocks, because the number could only affect the crack growth through the value of a.

In addition, on the assumption that the testing temperature in a series of experiments for the prediction does not significantly deviate from the temperatures in practical use, most material properties, such as Young's modulus, thermal conductivity, linear coefficient of thermal expansion, etc., can be approximated to be constant for the duration of substantial crack growth which will occur at the time of maximum thermal stress. Crack velocity, however, will not always be approximated in the same way, because it changes rapidly with temperature for materials with high Q. Thus, as for T in the exponential term, it would be desirable to take the variation of temperature with that of thermal shock severity into consideration. In addition, when the heating or cooling conditions are kept unchanged except for the temperature difference, σ_T is directly proportional to the temperature difference, ΔT,

$$\sigma_T = \Delta T \cdot f(t) \qquad (4)$$

where $f(t)$ is a function of time.

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On making the assumptions mentioned above, the time of the maximum stress is common for any value of ΔT, and the approximated temperature, T, over the duration of substantial crack growth can be expressed as follows:

$$T = T_0 + \alpha \cdot \Delta T, \qquad (5)$$

where T_0 is the base temperature before ΔT is given and α is constant. On these assumptions, Equation 3 can be integrated, giving:

$$a_I^{(2-n)/2} - a_F^{(2-n)/2} = \left(\frac{n-2}{2}\right)(\Delta T)^n G(T_0 + \alpha \cdot \Delta T) \qquad (6)$$

$$G(T_0 + \alpha \cdot \Delta T) =$$
$$A \cdot \exp\left\{-Q/R(T_0 + \alpha \cdot \Delta T)\right\} \cdot \int_0^\omega Y^n f^n \mathrm{d}t,$$

where a_I and a_F are the crack lengths before and after one thermal shock occurs, and ω is the periodic time of the thermal shock.

For ceramics of low Q, or for a small variation in ΔT, Equation 6 becomes:

$$a_I^{(2-n)/2} - a_F^{(2-n)/2} = \left(\frac{n-2}{2}\right) \cdot (\Delta T)^n \cdot G(Tr). \qquad (7)$$

$G(Tr)$ has a similar meaning to $G(T_0 + \alpha \cdot \Delta T)$ in Equation 6, and Tr is a representative value of the temperature over the duration of crack growth in a series of experiments.

Thus, when a reaches a_c after N cycles of thermal stressing, the following equation is given from Equation 7,

$$a_i^{(2-n)/2} - a_c^{(2-n)/2} =$$
$$\left(\frac{n-2}{2}\right) \cdot N \cdot (\Delta T_N)^n \cdot G(Tr) \qquad (8)$$

where ΔT_N is the temperature difference under which the life time of the ceramic is N, and a_i is the initial crack length of the ceramic before N thermal stresses are given.

In Equation 8, $a_c \gg a_i$ and $n \gg 1$, on assuming the occurrence of crack growth in stage I in the v–K_I curve [4]. Then, the following approximation can be made:

$$a_i^{(2-n)/2} \doteq \left(\frac{n-2}{2}\right) \cdot N \cdot (\Delta T_N)^n \cdot G(Tr). \qquad (9)$$

TABLE I Dimension, chemical composition and physical properties of each specimen.

Specimen	Composition	Coefficient of thermal expansion $(\times 10^{-6})$	Thermal conductivity $(\mathrm{cal\,cm^{-1}\,sec^{-1}\,°C^{-1}})$	Density $(\mathrm{g\,cm^{-3}})$	Dimensions	
					Diameter (mm)	Height (mm)
Soda-lime-silica glass	SiO_2 Na_2O CaO MgO	9.5 $(30 - 380°\,C)$	0.0026	2.48	4	70
Pyrex glass	SiO_2 B_2O_3 Na_2O Al_2O_3	3.25 $(0 - 300°\,C)$	0.0024	2.23	3	70
Mullite	$3Al_2O_3 \cdot 2SiO_2$ K_2O	4.4 $(25 - 700°\,C)$	0.01	2.6	6	70
Y_2O_3 stabilized zirconia	ZrO_2 Y_2O_3	9.9 $(25 - 1000°\,C)$	–	5.4	8	70

From Equation 8 or 9, the following equation may be derived for ceramics having a common a_i [5].

$$\frac{N}{N'} = \left(\frac{\Delta T_{N'}}{\Delta T_N}\right)^n. \qquad (10)$$

Equation 10 relates the results obtained by relatively short-term testing to those in practice for ceramics having a common a_i. Individual ceramics, however, generally have different initial cracks which cannot be known before failure. Therefore, the formula can be applied only to the representative value of the ceramic group, such as median, mode or mean values.

For individual ceramics, a statistical treatment is needed. As well known, the short-time fracture strength of ceramics follows the Weibull statistics [6],

$$P = \exp\left[-V(\sigma_f/\sigma_0)^m\right] \qquad (11)$$

where P is the survival probability under the fracture stress σ_f, m is the Weibull modulus, σ_0 is a normalization constant, and V is the stressed volume. The fracture stress, σ_f, is related to a_i through the following equation:

$$\sigma_f = K_{IC} \cdot Y^{-1} \cdot a_i^{-1/2}, \qquad (12)$$

where K_{IC} is the critical stress intensity factor of the ceramic. Substitution of Equation 12 into Equation 11 gives:

$$P = \exp\left[-V(a_i/a_0)^{-m/2}\right], \qquad (13)$$

where

$$a_0 = (K_{IC} \cdot Y^{-1} \cdot \sigma_0^{-1})^2. \qquad (14)$$

Substituting Equation 9 into Equation 13 gives

$$\ln(-\ln P) = \ln V' + \frac{m}{n}\ln N + m\ln(\Delta T_N), \qquad (15)$$

where

$$V' = V \cdot a_0^{m/2} \cdot \left(\frac{n-2}{2}\right)^{m/(n-2)} \cdot$$

$$A^{m/(n-2)} \cdot \{G(Tr)\}^{m/n}.$$

Equation 15 relates statistically the results of the relatively short term tests of a group of ceramics to those of long-term practice.

For a common $P(=P_0)$, the following equation is derived from Equation 15 [5],

$$\frac{N}{N'} = \left(\frac{\Delta T_{N'}}{\Delta T_N}\right)^n. \qquad (16)$$

This is identical to Equation 10 which is derived for ceramics of common a_i. The identity is not unexpected, because a common value of P corresponds to a common value of initial crack length in a group consisting of a large number of ceramics.

According to Equation 15, $\ln(-\ln P)$ plotted against $\ln N$ gives a straight line for a given value of ΔT_N. The lines run parallel to one another, as the tangent of a line is of the common value, m/n. Thus, when one line is determined for a value of ΔT_N, another line can be given only by parallel shifting of the line. The separation in the ordinate between the two lines, corresponding to ΔT_N and to $\Delta T_{N'}$, is given as

$$m\ln(\Delta T_N/\Delta T_{N'}).$$

This feature of the ln $(-\ln P)$–ln N plots closely resembles the SPT diagram introduced by Davidge et al. [6]. The resemblance is not surprising, because Equation 8 is identical to that for cyclic as well as constant mechanical stress [6], in which the life-time, τ, and the stress, σ, are substituted by N and ΔT_N, respectively. Because of the resemblance, the diagram given from Equation 15 may be called a T–SPT diagram (thermal shock severity-probability-time diagram).

3. Experimental
To examine the validity of the above formulae, the following experiments were carried out.

3.1. Specimens
The specimens were made of soda-lime-silica glass, Pyrex glass, mullite and Y_2O_3 stabilized zirconia. Their dimensions and chemical compositions are given in Table I. The specimens were prepared from a long bar by cutting with a diamond blade. The edge of some mullite specimens was polished with 2000 grit silicon carbide paste. Other specimens were not polished.

3.2. Apparatus and experimental procedures
The apparatus is shown in Fig. 1. Nine specimens were charged in the specimen holder. They were transferred from the hot zone in an electric resistance furnace into water, and then returned to the hot zone. This cycle was repeated until the number of cycles reach a given value. The specimens were held in the hot zone for 30 min, and in water for 5 min. The time for transfer from the hot zone to water was about 2 sec and that from water to the hot zone about 16 sec.

The temperature of the hot zone was controlled to within $\pm 0.5°$ C. The temperature of water was within the range 13 to 17° C. The temperature difference in the formulae, ΔT, generally is taken as the difference between the temperature of the hot zone and that of the water. For water quenching, however, as water vapour is apt to envelop the specimen in water, it is desirable to take ΔT as that between the temperature of the hot zone and the boiling point of water, 100° C, as pointed out by Davidge et al. [7]. From these considerations, in the present paper, ΔT was taken as that between the temperature of the hot zone and that of water (base temperature = water tem-
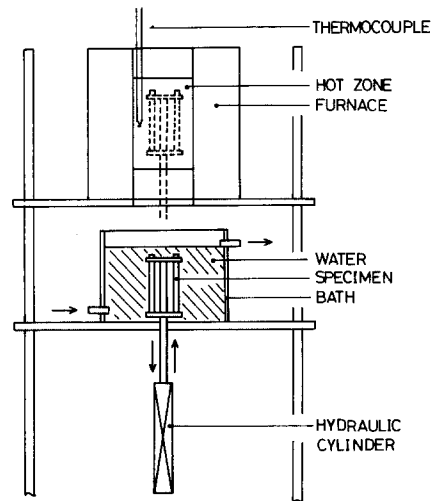
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Figure 1 Schematic diagram of the thermal fatigue testing apparatus.

perature) as well as that between the temperature of the hot zone and 100° C (base temperature = 100° C), and the results given for both systems of ΔT were examined.

The failure of a specimen was determined by the occurrence of a visible crack. Detection of the crack in a specimen was made at a given number of thermal cycles by use of dye.

The survival probability, P, was taken as the fraction given by dividing the number of specimens surviving after a given number of thermal cycles, by the total number of specimens, 9.

To estimate n and m by a mechanical method, the flexural strength was measured by three-point loading in which the span was 40 mm. The cross-head speed was varied from 0.5 to 50 mm min^{-1}. Nine specimens were used for each given value of cross-head speed. n was estimated from the dependence of the median of the flexural strength on the displacement speed, using the following equation [8],

$$\frac{\sigma_f}{\sigma_{f'}} = \left(\frac{\dot{\epsilon}}{\dot{\epsilon}'}\right)^{1/(n+1)} \tag{17}$$

where σ_f and $\sigma_{f'}$ are the flexural strengths at cross-head speeds of $\dot{\epsilon}$ and $\dot{\epsilon}'$, respectively. The Weibull modulus, m, was determined by applying the statistics to the flexural strength data.

4. Results and discussion
Cracks in the shocked specimens grew parallel to the specimen axis, except in zirconia and mullite
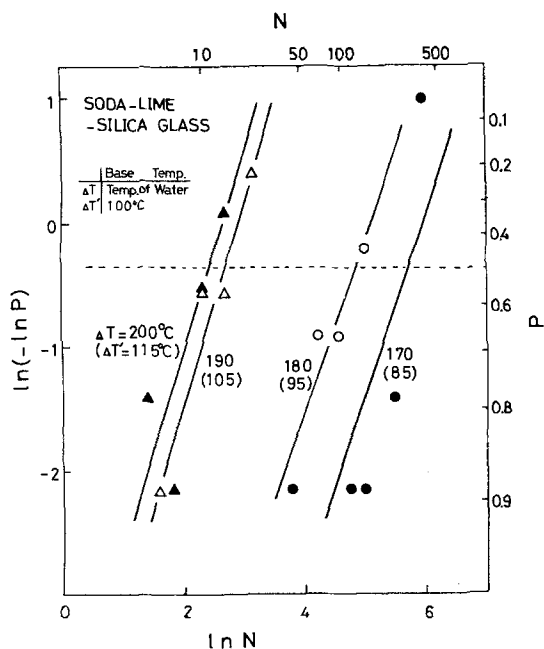
Figure 2 The survival probability, P, versus thermal fatigue life, N (T-SPT diagram), for soda-lime-silica glass. Parameters indicate the thermal shock severity, ΔT.
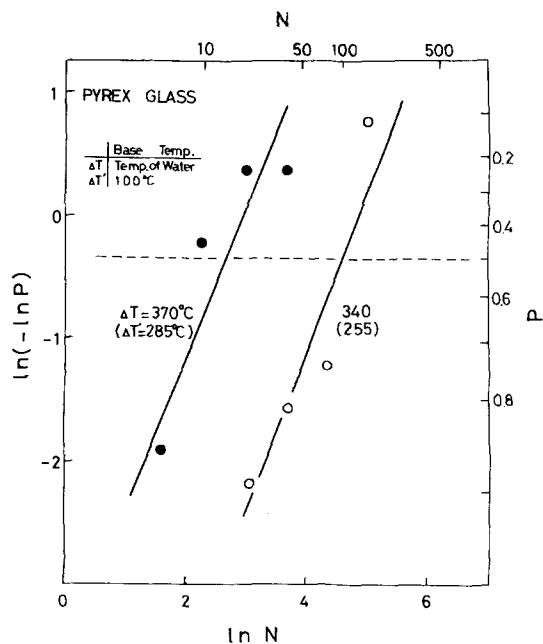

Figure 3 T-SPT diagram for Pyrex glass.

Figure 4 T-SPT diagram for mullite.

Figure 5 T-SPT diagram for Y_2O_3-stabilized zirconia.

specimens shocked with the temperature difference of $260°$ C (base temperature = water temperature), in which cracks perpendicular as well as parallel to the axis were observed. Little difference between polished and as-cut mullite specimens was observed.
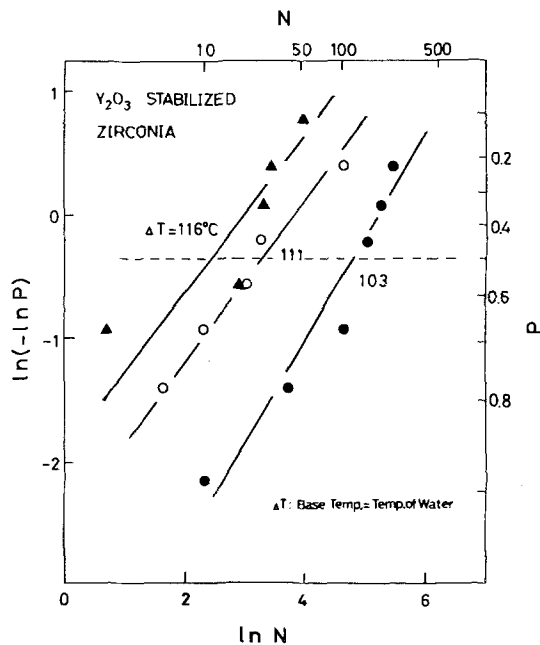
The survival probability, P, the fatigue life (thermal cycles), N, and the thermal shock severity of the two systems, ΔT_N, are plotted in Figs. 2 to 5 for each material. In these figures, $\ln\,(-\ln\,P)$ for a fixed value of ΔT_N in each system falls on a straight line except for the data for $\Delta T_N = 215°$ C in mullite. In addition, the lines run parallel to one another, except for mullite. These results show that Equation 15 is valid.

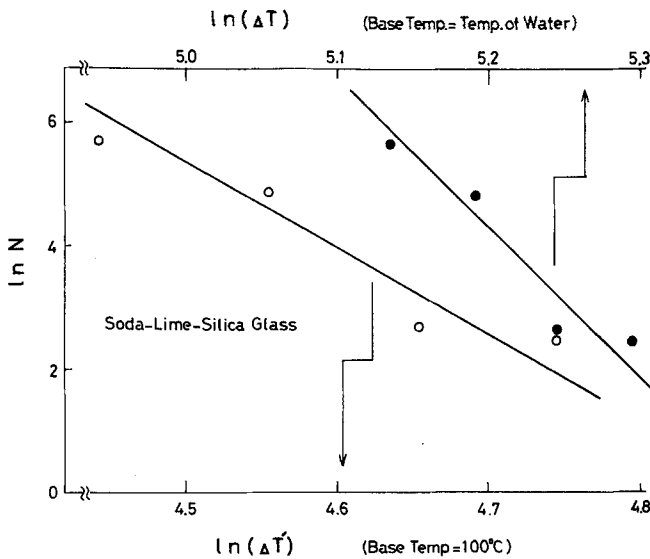*Figure 6* Thermal fatigue life, N versus thermal shock severity, ΔT, for soda-lime-silica glass. The data correspond to 50% survival fatigue life shown in Fig. 2. As the T-SPT diagram consists of parallel lines, the fatigue shown here shifts in parallel as the survival probability is changed.
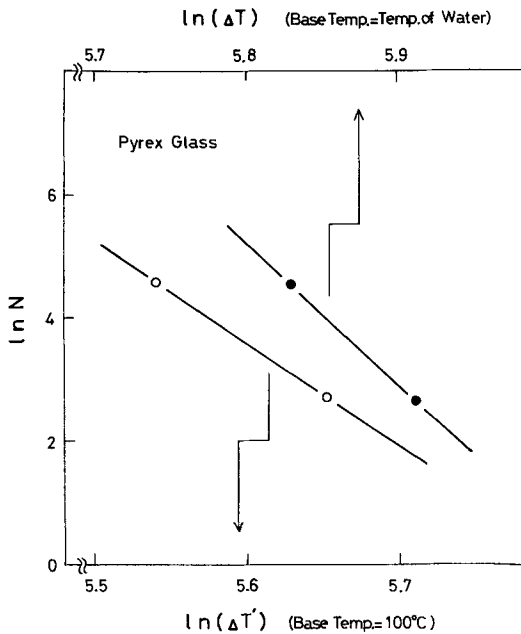

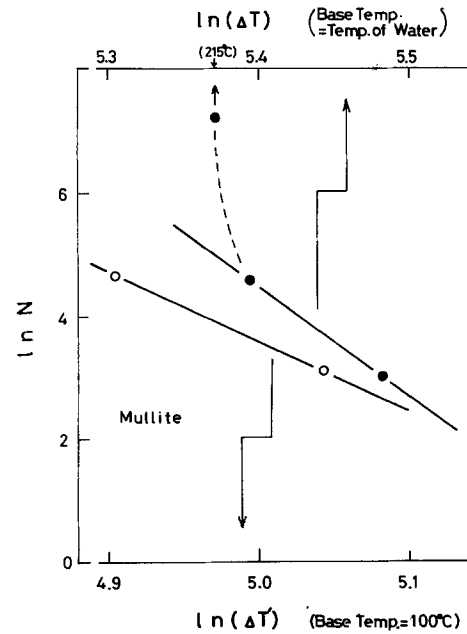*Figure 7* N versus ΔT for Pyrex glass (from Fig. 3).

Figure 8 N versus ΔT for mullite (from Fig. 4).

The value of ΔT is not so widely changed ($< 50°$ C), that the assumption of the small range of ΔT is satisfied. The validity of the formula partly results from the the small range of the experiment. The assumption may seem to limit the usefulness of the formula. However, this is hardly so because according to Equation 10 or Equation 15, for a prediction based on the results of a short term of the order of one hundredth of the practical term, only a small increment of about 23% in the temperature is needed for a ceramic with $n = 20$.

Moreover, the temperature at which the crack grows is always lower than the highest temperature. Therefore, the temperature variation in the crack growth is less than the variation of ΔT or that of the highest temperature in testing, and the assumed constancy of the material properties would be satisfied in many cases.

According to Equation 15 or 16, the constant, n, can be determined by plotting $\ln N$ against $\ln (\Delta T_N)$ for a fixed value of survival probability. The $\ln N$–$\ln (\Delta T_N)$ plots are given in Figs. 6 to
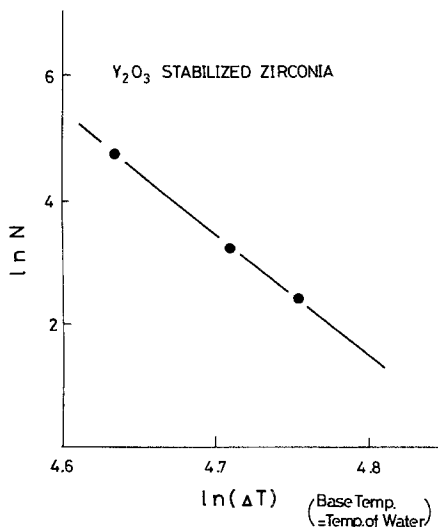
Figure 9 N versus ΔT for Y_2O_3-stabilized zirconia (from Fig. 5).

9 for individual materials. As seen in Fig. 6, the plots are linear for respective systems of ΔT_N, which shows the approximated formulae, Equations 15 and 10, to be valid. The value of n determined by application of Equation 10 or 15 to the figures are given in Table II in comparison with the values estimated from the mechanical strength data and those given in literature [9]. As seen in the figures or Table II, the value of n determined by taking the base temperature as that of water, is a little higher than that determined in other systems of ΔT_N. Their deviations from the literature values, however, are almost the same, although their signs are opposite. Thus, determination of a more suitable system from the two systems of ΔT_N was impossible in the present experiment. Since the difference in the two systems is small, for a

rough prediction either of the systems will give similar results.

The value of n determined from the figures agrees rather well with the literature values which refer to the stage I in the ln v–ln K_I curve, as mentioned above. The stress level of the specimen for the duration of crack growth was not measured; therefore, the value of n to be referred is not obvious, so long as some estimation is made. The crack velocity estimated from the fatigue life ($\gtrsim 20$ times) of soda-lime-silica glass is about 10^{-4} to 10^{-5} m sec^{-1}, on the assumption that the duration of crack growth is 0.5 to 5 sec for one thermal shock. The estimated value lies in the stage I of the glass [4]. Thus, the value of n to be referred will be that in stage I. Moreover, recently Soga *et al.* [10] found that the region of stage I extends, eliminating the region of stage II in the glass when the atmosphere consists of water vapour only. In the present experiment, the specimen is immersed in water and then heated in a furnace, so the space in the crack is thought to be almost filled with water vapour. Thus crack growth would occur in extended stage I even if the life time is much less or the crack velocity much higher. The possibility of the occurrence of crack growth in the region of stage II is also excluded by virtue of the following discussion. The integration of Equation 3 in stage II where n is zero, results in the conclusion that the life-time is almost independent of a_i or that it is almost the same for every specimen. But the actual life-time is not the same, thus it is disproved.

The time to rupture in the mechanical strength measurements was about 20 to 40 sec. From the crack velocity estimated from this life-time and the value of n estimated from Equation 17, crack

TABLE II The value of n determined from ln N–ln (ΔT) plots (Figs. 6 to 9) and other methods.

Material	From ln N–ln (ΔT) plots[*]		From ln σ_f–ln $\dot{\epsilon}$ plots[§]	Literature[¶]
	ΔT[†]	$\Delta T'$[‡]		
Soda-lime-silica glass	24	14	23	20
Pyrex glass	23	17	–	33
Mullite	18	11	19	–
Y_2O_3-stabilized zirconia	20	–	16	–

[*] See Equation 16.
[†] Temperature difference between the temperature of the hot zone and that of water.
[‡] Temperature difference between the temperature of the hot zone and 100° C.
[§] See Equation 17.
[¶] Obtained from Wiederhorn's original data [9].

growth can also be concluded to occur in the region of stage I. Thus, the rather good agreement between the values of n determined by different methods prove the validity of the formula.

By using the value of n obtained and applying Equation 15 to Figs. 2 to 5, the value of m for each material was determined. These values are given in Table III. With the exception of those for zirconia and mullite, the values at $\Delta T_N = 260°$ C are very large compared with those determined from flexural strength data as well as those in the literature [6]. This discrepancy is thought to be attributable to the fact that the crack growth in this experiment runs parallel to the axis, as mentioned before, in contrast to the perpendicular

TABLE III The value of m determined from ln $(-\ln P)$–$\ln N$ plots (Figs. 2 to 5) and the flexural strength data for each material.

Material	$\Delta T(°C)$	m	
		From T-SPT diagram*	From flexural strength data†
Soda-	170	36	8
lime-	180	34	
silica	190	41	
glass	200	41	
Pyrex	340	30	14
glass	370	38	
Mullite	220	32	8
	240	34	
	260	11	
Y_2O_3-	103	17	6
stabilized	111	13	
zirconia	116	13	

* Obtained by combination of n, determined by Equation 16, with m/n, determined from T-SPT diagram.
† See Equation 11.

growth under mechanical stressing. The statistics of the initial cracks in the perpendicular direction may not be identical to these in the parallel direction.

In specimens of zirconia and mullite subjected to a temperature change of $260°$ C, crack growth in the perpendicular direction occurs, which may be responsible for the rather good agreement in m with the value determined from the flexural strength data.

A plot of ln $(N(P = 0.5))$ against ln (ΔT_N), assuming $N(P = 0.5)$ as 1500 which is less than the actual value for $\Delta T_N = 215°$ C in mullite (Fig. 4), does not give a straight line, but a curve. From this result, Equation 16 evidently does not hold. Application of a formula similar to Equation 10 but derived from Equation 9 to the ln N–ln (ΔT_N) plots was tried, and a value of -1570 for n was obtained. The value of α' $(Tr = T_0 + \alpha'\Delta T)$ is estimated to be negative (~-1.3) assuming that Q for mullite is similar to that for soda-lime-silica glass $(Q \simeq 30 \, kcal \, mol^{-1})$ [11]. The negative values for n as well as α' cannot be accepted. Therefore, for interpretation of the result for $\Delta T_N = 215°$ C in mullite, introduction of some factors affecting the fatigue life, e.g. strengthening by moderately rapid cooling, seems to be needed, which is, however, left unknown.

Application of the formula to other authors' results was also made. Fig. 10 is drawn using the data for soda-lime-silica glass given by Hasselman et al. [5, 12]. A plot of ln N versus ln (ΔT_N) in Fig. 10 again gives a straight line, suggesting that Equations 10 and 15 are valid. The values of n determined from the tangent of the line are 30 and 16 (bath temperature = $65°$ C), 29 and 16
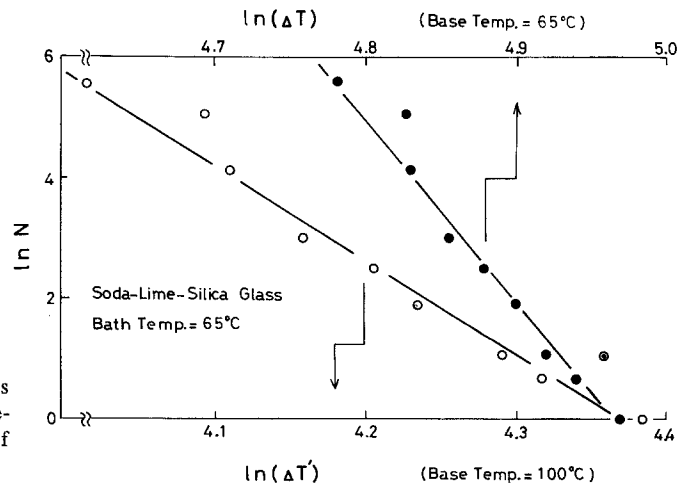

Figure 10 Thermal fatigue life, N, versus thermal shock severity, ΔT, for soda-lime-silica glass rods. (Plotted using the results of Hasselman et al. [12]).
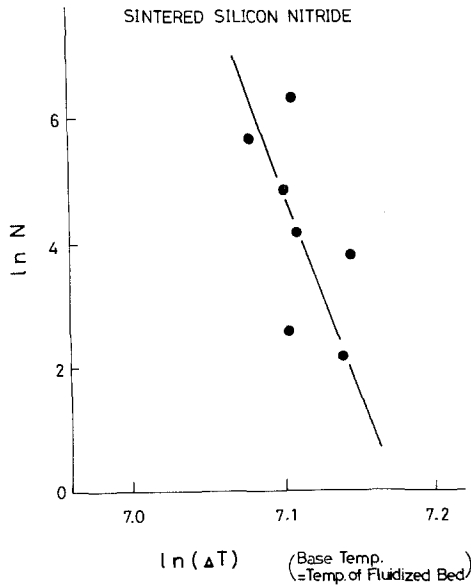
SINTERED SILICON NITRIDE

Figure 11 N versus ΔT for sintered silicon nitride. (Plotted using the results of Ammann *et al.* [13]).

(bath temperature $= 33^\circ$ C), for the system in which the base temperature is that of water and 100° C, respectively. These values, especially that for the system in which the base temperature is taken as 100° C, agree well with that (20) determined from the ln v–ln K_I plot obtained from the data given by Wiederhorn and Borz [9]. Fig. 11 was prepared using the data for sintered silicon-nitride given by Ammann *et al.* [5, 13]. Again a straight line is obtained with a tangent of about 60. This value agrees well with the value of 10 to 50 for the material [11]. This agreement in the value of n seems to prove the approximated formulae valid.

As seen above, Equation 15 or 16 gives a rough prediction of thermal fatigue life from a short-term test of ceramics. In the derivation of the formulae, the factor $G(T)$ is left unknown, which may appear to cause difficulty in applying the formulae to a proof test of the thermal fatigue life. However, it may be solved as follows: the mechanical strength of a ceramic, such as flexural or tensile strength, depends on its initial crack length, a_i. It increases as a_i decreases. Thus the initial crack length, a_{im}, corresponding to a median or a mode in the mechanical strength data will be common to that in the thermal fatigue data. In a short-time fracture experiment, the following relation is given from Equation 12,

$$K_{IC} = Y \cdot \sigma_{fm} \cdot a_{im}^{1/2}, \qquad (18)$$

where σ_{fm} is a median or a mode in the strength data. For thermal fatigue life, from Equation 9,

$$a_{im}^{(2-n)/2} = H \cdot N_m (\Delta T_t)^n, \qquad (19)$$

where N_m stands for the fatigue life corresponding to the median or mode, ΔT_t for the temperature difference of a fatigue test, and

$$H = \left(\frac{n-2}{2}\right) G(Tr). \qquad (20)$$

In Equation 19, ΔT_t is treated as a constant, therefore H is a constant.

Combination of Equations 18 and 19 gives the following equation for H:

$$H = \left(\frac{Y \cdot \sigma_{fm}}{K_{IC}}\right)^{n-2} \cdot N_m^{-1} \cdot (\Delta T_t)^{-n}. \qquad (21)$$

Substitution of Equation 21 into Equation 9 gives:

$$a_i^{(2-n)/2} \doteq \left(\frac{N}{N_m}\right)\left(\frac{Y \cdot \sigma_{fm}}{K_{IC}}\right)^{n-2}\left(\frac{\Delta T}{\Delta T_t}\right)^n. \qquad (22)$$

The initial crack length, a_s, which will prove the fatigue life longer than N_s under the thermal severity of ΔT_s, is given from Equation 22 as follows:

$$a_s^{(2-n)/2} \leqslant \left(\frac{N_s}{N_m}\right)\left(\frac{Y \cdot \sigma_{fm}}{K_{IC}}\right)^{n-2}\left(\frac{\Delta T_s}{\Delta T_t}\right)^n. \qquad (23)$$

Combining Equations 23 and 12 gives the proof test stress, σ_{fp}, which is related to the fatigue life, N_s, as follows:

$$\sigma_{fp} = \left(\frac{N_s}{N_m}\right)^{1/(n-2)}\left(\frac{\Delta T_s}{\Delta T_t}\right)^{n/(n-2)} \cdot \sigma_{fm}, \qquad (24)$$

or

$$\sigma_{fp} \doteq \left(\frac{N_s}{N_m}\right)^{1/n}\left(\frac{\Delta T_s}{\Delta T_t}\right) \sigma_{fm}. \qquad (25)$$

The ceramics having survived a mechanical stress of σ_{fp} will survive after N_s cycles of thermal shock severity of ΔT_s in practical use. The validity of the proof tests depends on the degree of accuracy in the correspondence of a_{im} in the mechanical strength data, to that in the fatigue life data. A sufficiently large number of specimens will improve the accuracy, and Equation 25 will be useful in practice.

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5. Conclusions

(1) Some formulae for the prediction of thermal fatigue life of ceramics are obtained.

(2) Experiments on some ceramics including glass show the validity of the formulae.

(3) A proof test for the life is suggested.

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